Transformer Terminal-Duality Model for Windings Turn-to-Turn Faults Simulation

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Abstract— Turn-to-turn fault (TTF) inception on a transformer can be very destructive and damaging. In order to check the performance of TTF protection algorithms, it is necessary to model the transformer in the presence of TTFs. Under minor TTF inception, the rise in the terminal currents of the transformer is just in the order of its no-load current. Therefore, in this study, a suitable transformer model should be employed to address the role of core and windings under the occurrence of minor TTF, accurately. This paper presents a new equivalent reluctance circuit for three-phase transformers in the presence of TTFs. This equivalent model employs the parameters determined by the finite element method. This model contains the admittance matrix and it is well-known as the terminal-duality model. For verification of the model, a real three-phase custom-built transformer is employed and modeled in this paper. The results obtained from the developed model are validated against the experimental results of TTF tests carried out on this transformer.

Keywords—finite element method, terminal-duality model, transformer model, turn-to-turn fault;

I. Introduction

Transformers occupy noticeable parts in power systems, being the essential links between generating stations and utilization points [1]. Failure of a transformer can cause irreversible and costly damage to the power grid. One of the most common reasons for transformer failures is its winding faults [2]. A turn-to-turn fault (TTF) starts with the degradation of the winding insulation between adjacent turns, but after a short time, it can lead to a severe phase to ground fault [3]. Recently, various methods have been introduced for fast detecting of TTFs [4-5]. However, the development and the validation of any TTF protection algorithms for a digital differential transformer protection require the precise determination of the transformer model [6]. The modeling of a healthy transformer or a transformer in the presence of TTF is fundamentally similar. The only difference is in the modeling of the faulted windings. The three main approaches for transformer modeling are as follow [7]:

a) Duality-based model: using the principle of duality, a transformer equivalent electric circuit can be derived from its magnetic circuit [8]. In these models, the flux balance, and consequently the non-linear core is accurately represented. However, their parameters are calculated, assuming that the magnetic field is in the axial direction in cylindrical geometry.

b) Leakage inductances-based model: parameters of these models obtained from short-circuit tests, and they usually use the inverse inductance matrix [9]. Although these models have an accurate short-circuit response, the non-linear core should separately include in the models. It is because the magnetizing effects get lost in the short circuit tests. Since in these models, the core is not considered in its actual position, the transformer excitation current is not modeled accurately.

c) Self and mutual inductances-based model: in these models, self and mutual inductances calculated by precise formula [10]. Due to the presence of iron-core, these inductances have very close values. Therefore, these models usually resulted in an ill-conditioned set of equations.

Fig. 1 represents the effects of transformer winding TTF on the magnetic flux intensity (\( \mathbf{H} \)) of a transformer in a finite element simulation software. Fig. 1(a) shows the axial magnetic fields of a healthy transformer. While, as it can be seen in Fig. 1(b), due to the winding fault occurrence, an intense radial leakage flux appears around the shorted turns. These leakage inductances cannot be calculated by a simple analytical formula based on the axial flux. For that reason, the duality-based model alone is not sufficient to model the TTFs of the transformers. Because of the inaccuracy of this formula in presence of radial flux, finite element models (FEMs) should be used for numerical computation of short-circuit admittances and leakage inductances.
In addition, the changes in the terminal currents of the transformer in case of minor TTF occurrence are insignificant [11], and it is comparable to the no-load current. Therefore, precise modeling of the core is important in the development of the TTF protection algorithms. Consequently, the leakage inductances-based model is not enough for accurate modeling of the transformer.

In this paper, a combination of the duality-based and leakage inductances-based models, called the terminal-duality model, is employed. This method uses the calculated admittance matrix and reluctance circuit of the transformer to model the transformer windings and core effects. By application of finite element simulations, all of the required parameters are determined. This model is explained in Section II. In Section III, a real three-phase custom-built transformer is modeled. The validity of the proposed model is verified, by comparing the simulated results against the custom-built transformer experimental measurements.

II. Terminal-Duality Model

The first step in modeling is to calculate leakage inductances \( L_{s,i,j} \) and short-circuit admittance \( Y_{i,j} \) of the transformer in the presence of TTF. With the application of these matrices, the inductance matrix of the transformer \( L \) can be calculated. The \( L \) matrix can model the terminal behavior of the transformer. Fig. 2 represents the whole terminal-duality model development process. As can be seen, \( Y \) and \( Ls \) can be calculated through finite element simulations. Also, based on the geometrical arrangement of core and windings, the equivalent reluctance circuit of the transformer is determined. The admittance- based terminal-duality model combines the core and windings model to determine an equivalent electrical circuit of the transformer, which can model the minor TTFs inception. The effect of zero-sequence flux is also considered in the model. This process is explained in detail in the following four steps.

A. Calculating \( Ls \) and \( Y \) by FEM

Leakage inductances can be calculated by the standard impedance voltage tests [12] and through the terminal measurements. In order to measure the short-circuit inductance between a pair of windings (for example, \( L_{s,i,j} \) between winding \( i \) and winding \( j \)), winding \( j \) must be short-circuited. While winding \( i \) is connected to a variable voltage source. Other windings remain open circuit. In the standard test, the voltage of source \( V_i \) must be limited enough for the rated current to pass through the winding \( i \). Overlooking the resistance, \( L_{s,i,j} \) can be calculated as [13]:

\[
L_{s,i,j} = \frac{V_j}{\omega L_i} \quad V_j = 0, I_k = 0 \quad k=1,..,n; k \neq i, j; j \neq i
\]  

Where \( I_i \) is the current of the winding \( i \), \( I_k \) is the current flowing through the other windings, and \( n \) is the number of windings per phase.

In addition, to measure the short-circuit admittance elements in row \( i \), winding \( i \) must be connected to a variable voltage source, and all of the other windings must be shorted. Similar to the latter test, the source voltage \( V_i \) must be selected in such a way to pass the rated current in the winding \( i \). Therefore, \( Y_{i,j} \) can be calculated as follows [14]:

\[
Y_{i,j} = \frac{I_j}{V_i} \quad V_i = 0 \quad k=1,..,n; k \neq i
\]  

The finite element method (a numerical method for solving partial differential equations), can be used to calculate the inductance parameters for complex winding configurations, easily and precisely. ANSYS Maxwell is a powerful software for finite element modeling of the electrical instruments. Since windings have cylindrical geometry, a 2D-FEM is defined in this software to calculate both \( Y \) (admittances) and \( LS \) (self-inductances) of the transformer. High accuracy is taken into account when defining transformer geometry. Moreover, the minimum size is chosen for the meshes, to reduce the computation error and ensure its accuracy. To comply with the simulation conditions with the standard terms of the test, the winding is directly injected by a current source \( I = I_{nominal} \). No longer, we need to use a variable voltage source to simulate the...
flow of rated current. An open circuit winding is modeled by a winding connected to a current source of \( I = 0 \). While a shorted winding is modeled by a winding connected to a voltage source of \( V = 0 \) with a very small series resistor \( (r = 0.0001 \text{ ohms}) \). The finite element simulations calculate the induced voltage and current in each winding. All the admittance elements and leakage inductances can be obtained through (1) and (2).

**B. Inductance Matrix Determination**

The short-circuit admittance matrix represents the relation between the phasors of transformer nodal voltage \( (V) \) and currents \( (I) \) as follows:

\[
Y_{bn} = I_n
\]  

(3)

However, the inductance matrix \( (L) \) is needed to determine the dynamic behavior of the transformer as:

\[
v(t) = L \frac{di(t)}{dt} + Ri(t)
\]  

(4)

Where \( i(t) \) and \( v(t) \) shows the instantaneous current and voltages of the transformer terminals. \( R \) is the terminal resistance, which can be calculated by a simple DC voltage test. Moreover, matrix \( L \) can be calculated in the terminal-duality model based on \( Y \) and \( L_s \). Terminal-duality models satisfy the branch-node transformation equations, which mean:

\[
V_b = Z_b I_b \Rightarrow I_b = Z_b^{-1} V_b
\]  

(5)

\[
V_b = AV_n \cdot A^T I_b = I_n
\]  

(6)

\[
I_n = A^T Z_b^{-1} A V_n
\]  

(7)

Where \( V_b \) and \( I_b \) are the branch voltage and current matrix, \( Z_b \) is the branch impedance matrix, and \( A \) is the incidence matrix. By comparing (7) with (3) and overlooking the resistive part of the branches, one can conclude that:

\[
Y_n = A^T (j \omega L)^{-1} A
\]  

(8)

Since \( A \) is not necessarily a square matrix all the time, usually \( L \) cannot be determined directly from (8). Therefore, some iterative solvers which can minimize the difference between the two sides of (8) should be used. The mutual inductance of \( L \) \( \left( M_{ij} \right) \) can be evaluated by the Matlab least-square routine, which is used for solving non-linear equations such as (8). Additionally, the self-inductance of \( L \) \( \left( L_0 \right) \) is directly obtained by the calculated leakage inductance in 2D-FEM simulations. As an example, for the transformer shown in Fig. 3, the starting point of the mutual inductance determination, using an iterative method, is given in (9) [13]:

\[
L_0 = \begin{pmatrix}
L_{1,2} & 0 & 0 \\
0 & L_{2,3} & 0 \\
0 & 0 & L_{3,1}
\end{pmatrix}
\]  

(9)

The numerically computed \( L_0 \) using FEM-calculated \( Y \) and \( L_s \), is sufficient for the development of the transformer terminal model. Since both \( Y \) and \( L_s \) are calculated or measured based on the short-circuit tests, this model has a precise short-circuit response.

**C. Geometrically-corrected core model**

Due to minor TTF inception on a transformer winding, a slight increase appears in their currents magnitudes; defined by differential currents, which are comparable with transformer excitation current. Therefore, when using any transformer model in presence of TTFs, iron-core should be modeled precisely. However, in short-circuit tests, the effects of the core are ignorable. As a result, \( L \) is not sufficient for precise modeling of the iron-core.

For precise modeling of the core effects and consequently no-load current, the reluctant circuit of the transformer must be determined. Also, to estimate the flux paths length, transformer geometry should be known. The core models must be added to the corresponding terminals of the determined winding model.

**D. Zero-Sequence Impedance**

Since the no-load current of the transformer can have a zero-sequence component, the zero-sequence impedance of the transformer should be calculated and implemented into the model. In the three-phase three-limb transformers, the zero-sequence flux circulates through the surrounding oil, air, fittings, and tank walls [15]. In order to measure the zero-sequence reactance, a voltage \( (V_0) \) is applied between the shorted line terminals of a star-connected winding and the neutral. The calculated zero-sequence impedance \( (V_0 / I_0) \) is shown with \( Z_0 \) in the model. Regardless of how the \( Z_0 \) effect is reflected in the model, the total zero-sequence path impedance must be equal with the measurement result.

**III. TTF Modeling of the Custom-built Transformer**

A custom-built three-phase three-leg stacked-core transformer is specially designed and manufactured for TTF tests and measurements. Dimensions of this 2 kVA, 400/400 V transformer, are presented in Fig. 3. Each phase \( (LV \) or \( HV \)) of this custom-built transformer consists of two separate windings that externally connected in series. First winding (with height = 165 mm) has 150 number of turns, while the other one has 50 turns. In order to generate minor \( (1\%) \) TTF, an externally accessible tap is placed on the winding, as shown in Fig. 3.

In order to model the transformer in the presence of TTF, the healthy phases and faulted phase should be modeled separately. To build a reluctant circuit for the transformer, the flux paths in the transformer window should be established. Then, assigning an inductance to each path, the duality model of the transformer can be developed.
A. Healthy Phase Modeling

Fig. 4 shows the reluctance circuit for a healthy phase of the transformer, including core and air paths. As can be seen, windings have a simple series configuration. There are only three flux paths between the windings. Therefore, the duality model of a healthy phase consists of a set of three mutually coupled inductances. In this case, as explained, three leakage inductances must be determined through 2D-FEM simulations (shown in Fig. 4).

Fig. 5 represents the duality model for a healthy phase, which has four windings. As can be seen, based on the existence of symmetry in the geometry and for the sake of simplicity, just one side of a leg is modeled. To model the flux paths through the core, four non-linear inductances are employed in the model. The leakage inductances are calculated using FEM simulations. It should be mentioned that all inductances are referred to an arbitrary number of turns (N₀ = 150).

The next step is to calculate the matrix Y for the healthy phase. The results of the 2D-FEM simulations are as follows:

\[
Y = \begin{bmatrix}
12.0976 & -12.2445 & -0.1789 & 0.3257 \\
-12.2445 & 21.1570 & -8.5247 & -0.3888 \\
-0.1789 & -8.5247 & 17.2674 & -8.5626 \\
0.3257 & -0.3888 & -8.5626 & 8.6249
\end{bmatrix}
\]  

(10)

According to Fig. 5, the matrix A can be obtained as:

\[
A = \begin{bmatrix}
1 & -1 & 0 & 0 \\
0 & 1 & -1 & 0 \\
0 & 0 & 1 & -1
\end{bmatrix}
\]  

(11)

Also, L₀ is determined (in μH) as follows:

\[
L₀ = \begin{bmatrix}
263.4344 & 0 & 0 \\
0 & 363.2253 & 0 \\
0 & 0 & 369.4529
\end{bmatrix}
\]  

(12)

This calculated matrix is the model of a healthy phase of the transformer. A similar process can be followed for the faulted phases.

B. Faulted Phase Modeling

As it can be seen in Fig. 3, a fault between any two turns divides the winding into three separated windings: two healthy windings and a faulted one. (While a fault between a winding turn and the ground divides the faulty winding just into a healthy winding and a faulted one.) Fig. 6 demonstrates the reluctance circuit of the faulted phase of the transformer. As can be seen, the arrangement of the windings is not as simple as in the healthy phases. There are eight flux paths between the windings. Thus, the duality model of a faulted phase should contain a set of seven mutually coupled inductances. In this case, seven leakage inductances must be measured by 2D-FEM simulations. These leakage inductances are L₆₁, L₆₂, L₆₃, L₆₄, L₆₅, L₆₆, and L₆₇ as shown in Fig. 7. These leakage inductances are diagonal and nonzero elements of the matrix L₀'. Respectively. The results of 2D-FEM simulations for calculating Y' are reported as (14). Also, the incidence matrix, according to Fig. 7 can be obtained as (15).
Matlab’s least-square routine can calculate the inductance matrix of the faulted phase. The results, as can be seen in (16), are reported in \( \mu \text{H} \).

\[
Y^\prime = \begin{bmatrix}
12.4835 & -12.9100 & 0.3519 & 0.2898 & 0.1006 & -0.3164 \\
-12.9100 & 22.3048 & -9.4364 & -0.4023 & -0.1451 & 0.5889 \\
0.3519 & -9.4364 & 17.9836 & -5.3422 & -0.2074 & 3.3500 \\
0.2898 & -0.4023 & -5.3422 & 3.6686 & -0.0545 & 1.8404 \\
0.1006 & -0.1451 & -0.2074 & -0.0545 & 0.5886 & -0.2822 \\
-0.3164 & 0.5889 & -3.3500 & 1.8404 & -0.2822 & 1.5192
\end{bmatrix}
\]

Adding zero-sequence impedance and core models according to the core dimensions, the complete model of the transformer is obtained. Fig. 8 represents the equivalent electrical circuit for the transformer that can model the TTF inception. As can be seen in Fig. 8, zero-sequence impedances are depicted in green while the non-linear core models have a blue color.

**IV. Proposed Model Verification**

In order to verify the model of the custom-built transformer (represented in Fig. 9), the simulated currents are compared with the results of experimentally tests carried out on the transformer.
The model are validated against the experimental results under employed and modeled in this paper. The results obtained from Moreover, leads to a more accurate topologically phase transformers in the presence of TTF is introduced, which reported in Fig. 10(e). As 

currents, the differential current of the faulted phase is separately of its (a) measured (b) simulated; differential currents: ((c) measured (d) simulated), and (e) differential current of the faulted phase.

Fig. 9 represents the experimental setup utilized in these tests. As can be seen, the setup consists of a 5kVA auto-transformer, a purely resistive three-phase load, the custom-built transformer, six current transformers, two digital storage scopes, and a switch. TTFs can be applied to the transformer through the switch and a fault resistor.

As shown in Fig. 8, windings of the transformer are connected externally in a way that each phase (LV/HV) has two windings (200 number of turns in total). The Yn-Yn connected transformer supplies a purely resistive. A TTF is implemented between two consecutive turns (1%) through a switch. Fig. 10 represents the simulated and measured primary currents and differential currents of the transformer during the TTF inception. During a TTF inception, secondary currents remain unchanged. Therefore, the secondary currents are not reported in Fig. 10. For a better comparison of the measured and the proposed model currents, the differential current of the faulted phase is separately reported in Fig. 10(e). As it can be seen, the transformer in the presence of TTF is modeled with high accuracy.

v. Summary and Conclusion

In this paper, a new equivalent reluctance circuit of three-phase transformers in the presence of TTF is introduced, which leads to a more accurate topologically-corrected model. Moreover, a real three-phase custom-built transformer is employed and modeled in this paper. The results obtained from the model are validated against the experimental results under transformer TTF tests. The validity of the model is verified by comparing the simulated currents against the measured current under the same TTF tests.

In addition, some crucial points that must be considered for accurate modeling of the transformer and TTF studies are: i) The distribution of the magnetic flux inside the transformer window fundamentally alters due to TTFs inception. Therefore, the simple analytical formulas have not sufficient accuracy for leakage inductance calculations. It seems that it is necessary to use the finite element model for minor TTF modeling. ii) Under minor TTF inception in a transformer, the magnitude of the no-load current will be comparable with the increase observed by the differential currents. Consequently, precise modeling of the core is vital. iii) Since zero-sequence impedance also affects the magnitude of the no-load current, it should be considered in its transformer modeling.

REFERENCES